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A new non-iterative method for pressure-driven snapshot simulations with EPANET

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Abstract

This study compares different recently proposed methods for Pressure-Driven snapshot simulations of water distribution networks using the EPANET software interface and proposes a new one. The proposed method is based on the insertion of a sequence of devices consisting of a General Purpose Valve (GPV), a fictitious junction, a reach with a check valve and a reservoir at each water demand node. The proposed method differs from other methods previously proposed in the literature – and similarly based on the insertion of sequences of devices consisting of a valve and a reservoir or emitter – in that it uses a GPV. In fact the GPV allows the user to define the relationship between the flow (i.e. supplied demand) and available pressure, making the proposed sequence of devices capable of representing different relationships among these variables, unlike the other non-iterative methods already proposed in the scientific literature, in which the relationship is implicitly fixed by the structure of the sequence of devices used. Applications to two case studies and comparison with the results of the non-iterative methods already proposed in the scientific literature highlight the accuracy and flexibility of the proposed method and show, by contrast, the unreliability and limits, in terms of precision, of some of the methods previously proposed in the literature.

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1. Introduction

Software programs for simulating the hydraulic behavior of pipe networks are of great importance for the design and management of water supply and distribution systems. Most of these programs are based on the global gradient algorithm – GGA ([1], [2]), which is also the solver code of EPANET [3], a software package that is widely used in a variety of settings, from the professional to the academic. In particular, in hydraulic simulation software based on the GGA proposed by Todini and Pilati [1], including EPANET, it is assumed that the *delivered flow* Q at each node with an unknown head is fixed and equal to the *required flow*, or water demand, Q^{req} at the node itself, whilst the flow in the pipes and the head at the nodes with an unknown head is assumed to be unknown. According to this assumption, therefore, the flow delivered at the i -th node is assigned and independent of the available head at the same node; this type of approach is generally referred to in the scientific literature as Demand-Driven (DD).

In reality, however, the flow delivered at the nodes of a network is tied to the available pressure head. This type of operation is referred to in the scientific literature as Pressure-Driven (PD) and the $Q-H$ link characterizing it can be expressed by the following relationship:

$$Q = \begin{cases} 0 & \text{if } H \leq H^{min} \\ \alpha Q^{req} & \text{if } H^{min} < H < H^{des} \\ Q^{req} & \text{if } H \geq H^{des} \end{cases} \quad (0)$$

where α is the coefficient enabling the flow rate to be modulated when $H^{min} < H < H^{des}$. Various formulations for characterizing α as a function of H , H^{min} and H^{des} have been proposed in the literature, including, for example, the ones proposed by Wagner et al. [4], Tucciarelli et al. [5] and Fujiwara and Ganesharajah [6], given respectively by:

$$Q = Q^{req} \cdot \left(\frac{H - H^{min}}{H^{des} - H^{min}} \right)^{0.5} \quad (0)$$

$$Q = Q^{req} \cdot \sin^2 \left(\frac{H - H^{min}}{H^{des} - H^{min}} \right) \quad (0)$$

$$Q = Q^{req} \cdot \frac{(H - H^{min})^2 \cdot (3H^{des} - 2H - H^{min})}{(H^{des} - H^{min})^3} \quad (0)$$

It is important to observe that in situations where a pressure deficit can occur – for example when assessing the reliability of a system after a pipe or a pumping station is shut off or taken out of service – it is advisable to use a PD simulator, which will make it possible to identify the nodes where the water demand is not completely (or not at all) met. In light of this, over the past decade a variety of approaches have been proposed which focus on the development of simulation models enabling a PD analysis of pipe networks. These approaches can be divided into two main types: the solver algorithm of the simulation model can be modified so as to take directly into account the relationship between delivered flow and available head at the node, or else a simulation of the PD type can be run using a DD simulation model, such as EPANET, after suitable adaptations have been made.

In the former case, algorithms have been developed which, by modifying the GGA method originally proposed in [1], enable the flow delivered at the nodes to be modified according to the available pressure (e. g. [7], [8]). Clearly, by their very nature, these approaches entail the implementation of new hydraulic simulation software.

In the latter case, by contrast, use is made of an already existing hydraulic simulation software program that operates in the DD mode. With specific reference to EPANET, which, as previously noted, is a DD software program whose use is well established, a number of techniques have recently been presented. Such techniques make it possible to carry out simulations with this type of software while achieving results that are equivalent to those provided by a PD

simulator. They can be essentially divided into three types. With the first type, based on the use of dynamic-link libraries (DLLs), it is possible to carry out PD simulations with EPANET by relying on a suitable programming environment (e.g. C, Visual Basic, MATLAB™), from which the DLL is called up; the second and third types enable PD simulations to be run directly using the executable software, i.e. the EPANET interface. More in details, the first technique consists in modifying the dynamic-link libraries (DLLs) of EPANET by introducing a variant of the “emitter” element originally available in the EPANET software program ([9]; [10]). In the case of the second technique, a PD simulation is carried out using the executable EPANET software (i.e. via the interface) along with a *manually iterative process* ([11]; [12]; [13]). This approach can clearly be used by an operator directly using the EPANET interface, but it is necessary to iterate the simulations by manually modifying the network structure each time in order to obtain the correct solution. With the third technique, the topological structure of the network is modified only once and a given sequence of devices, typically a reservoir or an emitter and a valve, are connected to each demand node. This solution makes it possible to obtain a PD solution of the network by directly using the executable EPANET software, via the interface, with a *single simulation*. Various possible sequences of devices have been recently proposed in the literature ([14]; [15]; [16]; [17]).

This study is focused on the latter type of approach and makes specific reference to the case of snapshot simulations. Below we will briefly characterize the different sequences of devices proposed in the scientific literature and formulate an original one based on the use of a valve that is of a different type from the ones already proposed and enables the water demand to be modulated based on the available pressure head and according to different relations, such as, for example, the ones proposed in [4] (see eq.(0)), in [5] (see eq.(0)) and in [6] (see eq.(0)). We will then analyze and discuss the accuracy, advantages and disadvantages of each of the methods with reference to two real-life cases and conclude with some final considerations.

2. Methods analyzed

The methods analyzed in this study for a PD solution of a pressurized network using the executable EPANET software (via the interface), in the case of a snapshot simulation, are the ones proposed by a) Bertola and Nicolini [14], b) Jinesh Babu and Mohan [15], c) Gorev and Kodzheshpurova [16], d) Sayyed et al. [17] and e) the new method proposed here. As previously observed, all of these methods can be used to carry out a PD snapshot simulation, via the EPANET interface, by connecting a specific sequence of devices to each demand node. In general, the devices used are a reservoir (R) or an emitter (E), several types of valves (e.g. Flow Control Valve (FCV), Pressure Reducing Valve (PRV)) and a pipe with a Check Valve (CV), the latter used to prevent reverse flow. The methods differ in terms of the devices considered to form the sequence to be connected to each node. Each method is presented in detail below.

In order to quantitatively show the modulation of the flow rate obtained with the methods, the description of each is immediately followed by a numerical example based on a very simple scheme consisting of a generic demand node n characterized by a water demand corresponding to $Q^{req} = 1$ L/s, a pressure head value H^{des} above which the water demand is fully met, equal to 40 m, and a pressure head value H^{min} equal to 10 m, below which no flow is delivered. For representing the modulation of the flow rate Q at the node, it is assumed that the pressure head H at the node n ranges between 0 and 50 m. From an operational standpoint, in EPANET, the variation in the pressure head H of the node n was obtained by setting the elevation z_n of this node at 0 m, connecting it to a reservoir R_{ref} with a pipe of negligible length and resistance, and setting the reservoir R_{ref} at an elevation ranging between 0 m and 50 m.

2.1. Method PRV – E

In the method proposed by Bertola and Nicolini [14], a sequence of devices composed of a PRV and an emitter (Figure 1a) is added at each demand node n . The demand at the node n is set equal to zero. The emitter has an elevation $z_e = z_n + H^{min}$, where z_n is the node n elevation, a value of the exponent $\gamma = 0.5$ (where the emitter delivers flow according to relation $Q = cH_e^\gamma$, being H_e the pressure head at the emitter) and a discharge coefficient $c = Q^{req} / (H^{des} - H^{min})^\gamma$. Finally, for the PRV a set value of $(H^{des} - H^{min})$ is assumed. It should be kept in mind that, by virtue of continuity, the flow delivered from the emitter will correspond to the delivered/outgoing flow from the

node n . Thus, if the pressure head at the node n is $H \leq H^{\min}$, the PRV will not permit reverse flow from the emitter to the node (in EPANET the PRV also works like a check valve - CV) and the outflow from the node n is zero; if the pressure head at the node n is $H^{\min} < H < H^{\text{des}}$, the emitter will release flow according to the following relation:

$$Q = \frac{Q^{\text{req}}}{(H^{\text{des}} - H^{\min})^\gamma} \cdot (H - H^{\min})^\gamma \quad (0)$$

being the pressure head H_e at the emitter equal to $H - H^{\min}$ (being $z_e = z_n + H^{\min}$). Finally, if the pressure head H at the node n is $H \geq H^{\text{des}}$, and thus the pressure head H_e at the emitter is $H_e \geq (H^{\text{des}} - H^{\min})$, the PRV will limit the pressure head H_e of the emitter to the fixed value $(H^{\text{des}} - H^{\min})$, so that the delivered flow does not exceed demand. The graph in Figure 2a shows the pattern of Q/Q^{req} as a function of H with the PRV – E method. In particular, it should be noted, that eq. (0) represents the second case of eq.(0) if α is expressed with the expression of Wagner et al. [4] (see eq.(0)).

2.2. Method FCV – CV₀ – R

In the method proposed by Jinesh Babu and Mohan [15], a sequence of devices composed of a FCV, a fictitious junction n_f , a reach with a CV and a reservoir R are added at the demand node n (Figure 1b). The demand at the node n is set equal to zero, whilst the flow delivered from the node will correspond to the discharge through the above-mentioned sequence of devices in the direction of the reservoir. A set value equal to Q^{req} is assumed in the FCV to prevent the delivered flow from exceeding demand; the fictitious junction n_f and the reservoir R are placed at an elevation $z_R = z_n + H^{\min}$; the reach with check valve is characterized by negligible head losses and this is emphasized here by using the subscript “0” associated with the check valve symbol, i.e. CV₀. In the case of pressure heads lower than H^{\min} at the node n , the CV₀ will not permit the passage of flow from the reservoir to the node n ; where $H \geq H^{\min}$, the discharge would tend to grow in an unlimited manner, but the FCV limits the value to Q^{req} . The graph in Figure 2a shows the pattern of Q/Q^{req} as a function of H with the FCV – CV₀ – R method. It is worth noting that in this case no modulation of flow rate is observed. It is worth noting, moreover, that in some numerical applications proposed by Jinesh Babu and Mohan [15], there are values of Q ranging between 0 and Q^{req} at some nodes when $H = H^{\min}$. This “apparent” modulation of the flow rate depends solely on the upstream network characteristics, and in particular on the heads at the nodes with an imposed head and the head losses in the pipes upstream of the demand node considered, *but not on the head H at the node considered*. A technique substantially identical to the one proposed by Jinesh Babu and Mohan [15] was presented by Sivakumar and Prasad in [18]; they added a reach with a CV between the demand node n and the FCV, a solution that does not introduce any change compared to the method discussed above.

2.3. Method FCV – CV_{ml} – R

In the method proposed by Gorev and Kodzheshpirova [16] a sequence of devices composed of a FCV, a fictitious junction n_f , a reach with a CV having significant minor losses (hereinafter CV_{ml}, where “ml” stands for “minor loss”) and a reservoir R are added at the demand node n (Figure 1c). The demand at the node n is set equal to zero. A set value equal to Q^{req} is assumed in the FCV to prevent the discharge from exceeding the water demand; the fictitious junction n_f and the reservoir R are set at an elevation $z_R = z_n + H^{\min}$; the reach with CV_{ml} must be characterized by negligible friction losses but must have a minor loss characterized by a coefficient ζ_{CV} equal to:

$$\zeta_{CV} = 2g \frac{H^{\text{des}} - H^{\min}}{\left(\frac{Q^{\text{req}}}{\pi d_{CV}^2 / 4} \right)^2} \quad (0)$$

being d_{CV} is the diameter of the reach with the check valve CV_{ml} .

Where $H < H^{min}$, the reach with CV_{ml} will not permit flow from the reservoir to the node n ; where $H^{min} \leq H < H^{des}$, the reach with CV_{ml} will allow modulation of the flow rate (towards the reservoir) thanks to the minor loss; finally, when $H \geq H^{des}$ the FCV will prevent the delivered flow from exceeding Q^{req} . In greater detail, in this case the flow rate modulation mechanism depends on the motion equation associated with the sequence of devices itself. In particular, given the minor loss coefficient ζ_{CV} provided in eq.(0) it can be demonstrated that the flow rate modulation obtained with this method reproduces the second case of eq.(1) if α is expressed using the expression of Wagner et al. [4], as with the PRV – E method (see also Figure 2a).

2.4. Method FCV – CV₀ – E

In the method proposed by Sayyed et al. [17], a sequence of devices composed of a FCV, a fictitious junction n_f , a reach with a CV without minor losses (hereinafter CV_0) and an emitter E are added at the demand node n (Figure 1d). The demand at the node n is set equal to zero. In the FCV, Q^{req} is fixed so as to prevent the discharge from exceeding the water demand; the fictitious junction n_f and the emitter are set at an elevation $z_e = z_n + H^{min}$. The emitter delivers flow according to relation $Q = cH_e^\gamma$ and γ , according to Sayyed et al. [17], must take on a value equal to $\gamma = 2/3$. With this configuration, where $H < H^{min}$, the reach with CV_0 will not permit flow from the emitter to the node n , whilst where $H^{min} \leq H < H^{des}$, the emitter will modulate the flow rate according to relation (2) and, finally, when $H \geq H^{des}$, the FCV will prevent the delivered flow from exceeding the water demand Q^{req} . In Figure 2a the pattern of Q/Q^{req} as a function of H for both $\gamma = 2/3$ and $\gamma = 0.5$ are shown, recalling that case of $\gamma = 2/3$ is the one originally proposed by Sayyed et al. [17], whereas with $\gamma = 0.5$, the flow rate modulation obtained with this method reproduces the second case of eq.(1) if α is expressed using the expression of Wagner et al. [4], as with the PRV – E and FCV – CV_{ml} – R methods.

2.5. Proposed Method: GPV_{WTF} – CV₀ – R

The method proposed in this paper entails adding a sequence of devices composed of a General Purpose Valve (GPV), a fictitious junction n_f , a reach with a CV without minor losses (hereinafter CV_0) and a reservoir R at the demand node n (Figure 1e). The demand at the node n is set equal to zero. The fictitious junction n_f and the reservoir are set at an elevation $z_R = z_n + H^{min}$; in the reach with CV_0 , the friction head losses (as well as minor losses) must be negligible. A head loss curve is fixed for the GPV; this curve represents the relation between the discharge Q through the valve and the head loss ($H - H^{min}$) induced by the valve itself (in fact, taking into account that upstream of this valve, that is, on the node side, the head is $z_n + H$ whereas downstream of this valve, that is, on the reservoir side, the head is $z_R = z_n + H^{min}$). The curve must have a strictly monotonically increasing pattern so that each discharge value will have only one head loss value associated with it and vice versa. Practically speaking, the head loss curve characterizing the GPV is constructed as a continuous piecewise function:

$$Q = \begin{cases} f(H) & \text{if } H^{min} \leq H \leq H^{des} \\ Q^{req} \cdot \left(1 + \varepsilon \cdot \frac{H - H^{des}}{H^{des}} \right) & \text{if } H > H^{des} \end{cases} \quad (0)$$

where $f(H)$ can be represented, for example, by the relation of Wagner et al. [4] (see eq.(0)), Tucciarelli et al. [5] (see eq.(0)) or Fujiwara and Ganesharajah [6] (see eq.(0)) (see Figure 2b). If $H > H^{des}$, the function characterizing the head loss curve, in accordance with eq.(0), should be constant, i.e. the discharge Q should be equal to the demand Q^{req} irrespective of the head H ; but, since the head loss curve must have a strictly monotonically increasing pattern, as previously observed, in order to numerically overcome the problem it is assumed that where $H > H^{des}$ the discharge will increase infinitesimally, or in any case in a negligible manner from an operational standpoint, according to the second relation of eq. (7), being ε a small magnitude selected at will (for example in the order of 10^{-5}). With the proposed sequence of devices, therefore, in the case of $H < H^{min}$ (remembering that the head H refers to the node n),

the reach with CV_0 will not permit flow from the reservoir to the node n and thus the discharge Q will be zero; where $H^{min} \leq H < H^{des}$, the GPV will modulate the flow rate according to the relation described by the first part of the function characterizing the head loss curve (see eq.(0)); finally, where $H > H^{des}$, the delivered flow will remain practically constant and equal to Q^{req} in accordance with the second part of the piecewise function characterizing the head loss curve. In Figure 2a the pattern of Q/Q^{req} as a function of H assuming different expressions for the function $f(H)$ of eq.(0), i.e. the relation of Wagner et al. [4] (GPV_w), Tucciarelli et al. [5] (GPV_T) and Fujiwara and Ganesharajah [6] (GPV_F), are shown. It is evident that the advantage of this approach compared to the previous ones lies in the possibility of using any relation whatsoever between the delivered flow Q and the value $(H - H^{min})$; this introduces a significant novelty compared to the methods presented in the literature to date.

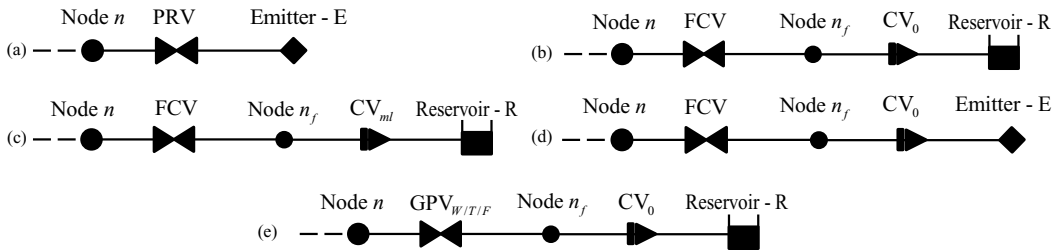


Fig. 1. Sequences of devices characterizing the analyzed methods: (a) PRV – E, (b) FCV – CV₀ – R, (c) FCV – CV_{ml} – R, (d) FCV – CV₀ – E, (e) GPV_{w/T/F} – CV₀ – R.

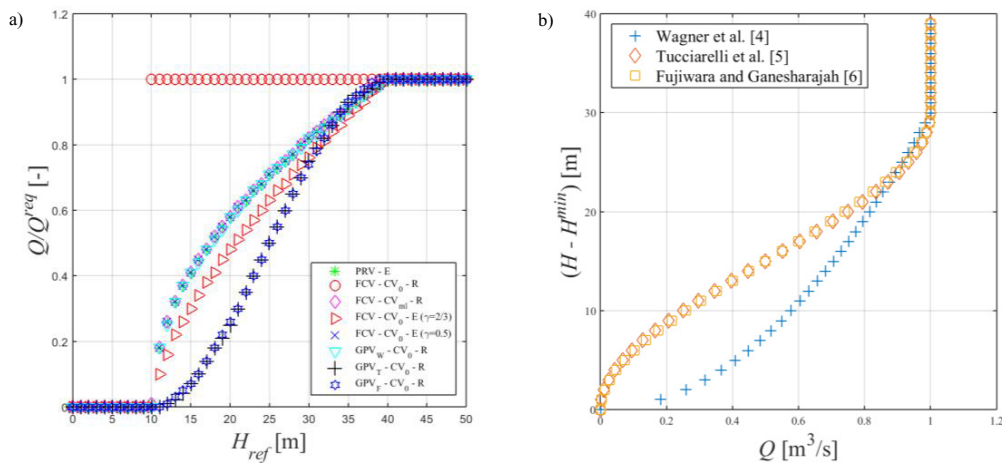


Fig. 2. (a) Pattern of Q/Q^{req} as a function of H_{ref} for the different methods analyzed; (b) Head loss curve for the GPV, obtained using the relations of Wagner et al. [4], Tucciarelli et al. [5] and Fujiwara and Ganesharajah [6].

3. Case studies

Two real-life cases were used to compare the analyzed methods. The network used in the first case study was taken from an article by Estrada et al. [19]; it is a network with a tree-like structure fed by one reservoir and equipped with a pressure sustaining valve (PSV) immediately downstream of the tank/reservoir. The second case consists in the network of the historical centre of a city in northern Italy, which extends for an overall length of about 72 km and serves approximately 60,000 people; it is characterized by a homogeneous elevation of the nodes and two reservoirs likewise set at the same elevation, and was taken from [7]. The values of the pressure heads H^{min} and H^{des} were

assumed equal to 25 m and 30 m, respectively, at all nodes. In both the case studies, *related to snapshot simulations*, the results obtained with the analyzed methods were compared with the values obtained using the pressure-driven algorithm developed by Alvisi and Franchini [7] using the formulations expressed by eqs.(0), (0) and (0), so as to compare each analyzed method with the PD algorithm using the same relation between nodal delivered flow Q and nodal pressure head H .

4. Discussion of the results and conclusions

The first case considered is a network featuring a PSV downstream of the tank. Given that this network has already been analyzed in the literature in order to assess the conflicts that can arise as a result of the interaction between a PSV and the use of sequences of devices, it is addressed again in this article to verify the compatibility of the methods analyzed here with the use of a PSV. It was seen that, when the PRV – E [14] and FCV – CV₀ – R [15] methods were used, EPANET failed to achieve convergence (at the end of the simulation the warning “System Unbalanced” appeared even when 1000 iterations are carried out, versus the 10-15 required by EPANET for the solution of the network modified with the other methods); the remaining methods, on the other hand, showed to be compatible with the use of the PSV and EPANET converged to solutions that were very similar each-other and characterized by mean and maximum errors, as compared to the results output by the PD algorithm [7], in the order of 0.02 L/s and 0.2 L/s (corresponding to percentage errors around 0.04% and 0.7%) respectively.

In the second case study, the elevation of the tanks was varied from H^{des} to H^{min} . Table 1 shows the corresponding mean, minimum and maximum absolute and percentage errors. In particular, it can be observed that in this case the FCV – CV₀ – R method generally gives larger errors than the other methods as a result of the inability of the method to modulate the flow rates between 0 and Q^{req} according to a pre-established relation; the maximum error, equal to about 13 L/s, occurs at the nodes with head values close to but slightly greater than H^{min} , for which the FCV – CV₀ – R method gives a delivered flow equal to Q^{req} , whereas all the other methods give a delivered flow close to 0 and in any case governed by the modulation relation. As far as the other methods are concerned, it can be observed that the FCV – CV_{ml} – R method [16] also gives a fairly large error, equal to almost 6 L/s. This error occurs at a specific node of the network with a head value close to H^{min} . All the other methods produce mean, minimum and maximum absolute and percentage errors that are quite similar each-other.

Table 1. Case Study III: Mean, minimum and maximum absolute and percentage (within brackets) errors obtained using the various methods versus the PD algorithm formulated by Alvisi and Franchini [7].

Method	Mean error [L/s] – [%]	Minimum error [L/s] – [%]	Maximum error [L/s] – [%]
PRV – E	0.0221 – (0.31)	0.00 – (0.00)	0.1602 – (0.64)
FCV – CV ₀ – R	1.6658 – (27.59)	0.00 – (0.00)	13.011 – (52.05)
FCV – CV _{ml} – R	0.1467 – (2.25)	0.00 – (0.00)	5.9285 – (23.71)
FCV – CV ₀ – E	0.0221 – (0.31)	0.00 – (0.00)	0.1602 – (0.64)
GPV _w – CV ₀ – R	0.0230 – (0.33)	0.00 – (0.00)	0.1702 – (0.68)
GPV _r – CV ₀ – R	0.0309 – (0.56)	0.00 – (0.00)	0.3399 – (1.36)
GPV _f – CV ₀ – R	0.0213 – (0.29)	0.00 – (0.00)	0.1978 – (0.79)

In conclusion, with respect to snapshot simulations, it can be affirmed that the techniques FCV – CV₀ – E [17] and GPV_{w/T/F} – CV₀ – R (proposed method) are capable of correctly reproducing the functioning of a network in the Pressure-Driven mode, unlike the other techniques, which, when different real-life cases were considered showed a number of limits, specifically: the FCV – CV₀ – R technique [15] does not enable the flow rate to be modulated between 0 and Q^{req} according to a pre-established relation and proved to be unreliable where a PSV was present; the PRV – E technique [14] showed to be unreliable where a PSV was present; and the FCV – CV_{ml} – R technique [16] gave a large error for a nodal head value close to H^{min} .

The proposed technique, $GPV_{W/T/F} - CV_0 - R$, also has an advantage in that enables any $Q - (H - H^{min})$ relation to be established between discharge and head loss. Therefore, any relation between delivered flow and available head can be used without necessarily having to use the formulation of Wagner et al. [4] as in all the other methods analyzed.

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