1	Multiobjective Valve Management Optimization Formulations for Water Quality				
2	Enhancement in Water Distribution Networks				
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#### 25 Abstract

26 Abstract: Water distribution networks (WDNs) need to guarantee that water is delivered with 27 adequate quality. This paper compares the performance of 12 multiobjective procedures to limit 28 water quality deterioration in a WDN through the optimal operation of valves. The first objective 29 (ObF1) is to minimize the water age, chosen as a surrogate parameter of quality deterioration, and 30 the second objective (ObF2) is to minimize the number of valve closures. The 12 procedures are 31 derived from the combination of 4 different optimization algorithms and 3 formulations of ObF1, 32 namely, to minimize the maximum, the arithmetic mean, and the demand-weighted mean water 33 age. The optimization algorithms considered are random search (RS), Loop for Optimal Valve 34 Status Configuration (LOC), and a combination of each of these two with the Archive-based Micro 35 Genetic Algorithm. The procedures are tested on two networks of different complexity. Results 36 show how LOC is able to find near-optimal solutions using a fraction of the computational time 37 required by a brute force search. Furthermore, among the ObF1 formulations, the use of the 38 averages (either arithmetic or demand-weighted) gives better results in terms of impact on the 39 population served by a WDN.

40 **Keywords:** water distribution network; multi-objective optimization; valves operation; water age.

## 41 INTRODUCTION

42 Water distribution networks (WDNs) are commonly designed to meet future situations, such as 43 population growth and industrial development, or to handle extraordinary events, such as urban 44 fire. Therefore, utilities often have to manage oversized-pipe systems characterized by reduced 45 velocities and high water age, defined as the time required for a drop of water to travel from the 46 main delivery point to a consumer. An increment of water residence time can negatively impact 47 the microbiological quality of the potable water (USEPA 2002). In particular, a high age value 48 implies deteriorated water quality in terms of chlorine residual concentration reduction and of 49 disinfection byproduct (DBP) formation, which may have carcinogenic effects on human health.

50 This study proposes a methodology to optimally manage the operational status of valves to modify 51 a network configuration solving a multiobjective optimization (MOO) problem in order to reduce 52 water quality deterioration expressed in terms of age. Different techniques have been widely used 53 for optimizing WDN design and operation (Mala-Jetmarova et al. 2018). In WDN design, 54 optimization problems have been mainly formulated considering the minimization of construction and operational costs and the maximization of resilience or head pressure. For example, Cembrano 55 56 et al. (2000) adopted a generalized reduced gradient to minimize WDN operational costs, while 57 Giustolisi et al. (2012) addressed the same problem considering leaks and using evolutionary 58 optimization algorithms. Creaco et al. (2015) used a multiobjective approach to optimize design 59 and operation considering installation and operational costs as objective functions. For the efficient 60 operation of a WDN, optimization problems have been formulated mainly considering operating 61 cost minimization (e.g., Jamieson et al. 2007) and pump scheduling optimization (e.g., Castro 62 Gama et al. 2015).

63 Some works suggest optimizing WDN operation using valve management with different solvers 64 and for different purposes, including pressure control, backflow prevention, and sectorization for 65 demand control (e.g., Di Nardo et al. 2014). For instance, Jowitt and Germanopoulos (1992) 66 proposed optimal scheduling of pumps and valves to minimize energy consumption using linear 67 programming, while Carpentier and Cohen (1993) used discrete dynamic programming. 68 Minimization of operational costs by valve scheduling was solved by Ulanicki and Kennedy 69 (1994) using an augmented Lagrangian method. The same problem was also ad-dressed solving 70 one part using a projected gradient method and the other part by a complex method (Cohen et al. 71 2000a, b). While water quality has been taken into account only recently in the design of WDNs, 72 it has been often considered in the optimization of WDN operation, for example, through effective 73 booster disinfection (e.g., Boccelli et al. 1998) or considering the minimization of re-chlorination 74 costs (e.g., Ostfeld and Salomons 2006; Li et al. 2015). In optimization problems, water quality 75 has been considered either as objective (Fu et al. 2013; Shokoohi et al. 2017) or constrained (Bi 76 and Dandy 2014; Kanta et al. 2011; Andrade et al. 2016), in terms of either chlorine residual 77 concentration or water age.

Owing to the uncertainty related to the adoption of existing formulations and to the relative reaction coefficients used to model water quality parameters (for example, to predict DBP formation or chlorine decay), it is preferable to use a more general and less uncertain parameter 81 such as age, as has been done in other studies (Fu et al. 2013; Shokoohi et al. 2017). Instead of 82 using chlorine (Bi and Dandy 2014; Kanta et al. 2011; Andrade et al. 2016) or DBP concentrations 83 (Quintiliani et al. 2018), in this study water age is chosen as the parameter since many aspects of 84 water quality deterioration depend on it (Machell and Boxall 2014). Moreover, defining and 85 evaluating water age is not a trivial task. In this paper, water age is computed following the 86 common approach of estimating it as the flow-weighted average age value of merged flow at a 87 node, even if such an approach has some limitations. Other en-hanced approaches could be 88 adopted (Machell et al. 2009; Zhao et al. 2018) as alternatives to the presented methodology.

89 Depending on the flow velocities in the system, water age can be modified by varying the fluxes 90 through tank- level regulation, changing the network configuration using valves, or opening hy-91 drants to increase discharges. As in Prasad and Walters (2006), the methodology presented in this 92 paper minimizes water age by means of valve management. In fact, this option makes it possible 93 to intervene without losing a precious resource, and the valves can be reopened during critical 94 scenarios. Since reopening may cause the release of accumulated material, in the proposed 95 procedure their movements are intended as a long-term operation for the reconfiguration of the 96 fluxes in the network, and not necessarily as a real-time management procedure.

97 In Prasad and Walters (2006), the optimization of pipe closures to minimize residence time was 98 formulated as a single-objective problem solved using genetic algorithms. The novelty of the 99 presented contribution consists of three main aspects: first, the adoption of a multiobjective 100 optimization problem formulation, introducing a second objective function; second, the evaluation 101 of different optimization algorithms, from the simplest random search (RS) to the advanced 102 evolutionary algorithm Archive-based Micro Genetic Algorithm (AMGA2) (Tiwari et al. 2011); 103 third, the application of a new algorithm suitable for this specific problem, namely, Loop for 104 Optimal valve status Configuration (LOC). The same three objective functions proposed by Prasad 105 and Walters (2006) are evaluated, and their effectiveness is investigated. Considering 4 different 106 optimization algorithms (with the third and fourth ones being a combination of AMGA2 with RS 107 and LOC) and the 3 objective functions, 12 different procedures are obtained and compared. They 108 are applied to two distribution net-works of different complexity: the example network used by 109 Prasad and Walters (2006) and a real network system in Kentucky (Jolly et al. 2012).

110 The paper is structured as follows. First, the formulation of the optimization problem is presented

111 and then the general methodology is described. Next, the two considered networks are introduced,

followed by the analysis of results and discussion. Finally, conclusions are presented and futureworks discussed.

# 114 **DEFINITION OF THE OPTIMIZATION PROBLEM**

#### 115 **Objective functions**

116 Two objective functions are considered in the optimization problem formulation. The first one 117 (ObF1) aims to minimize water age at demand nodes, and the following three formulations are 118 explored one at a time (Prasad and Walters 2006):

Maximum Water Age, *MaWA*, represents the maximum age that occurs during the simulation period across all demand nodes:

121 
$$ObF1 = \min\{MaWA\} = \min\{\max\{WA_{i,t}\} \forall i = 1 \dots T_n, t = 0 \dots TST\}$$
 (1)

• Mean Water Age, *MeWA*, representing the arithmetic average of the ages at all nodes:

123 
$$ObF1 = \min\{MeWA\} = \min\left\{\frac{1}{T_n * T_{step}} \sum_{i=1}^{T_n} \sum_{t=0}^{TST} WA_{i,t}\}\right\}$$
(2)

Demand weighted Mean Water Age, *DeMeWA*, represents the average of the ages
 calculated assigning at each node a weight equals the demand requested at each time step:

126 
$$ObF1 = \min\{DeMeWA\} = \min\{\frac{\sum_{i=1}^{T_n} \sum_{t=0}^{TST} WA_{i,t} * q_{i,t}}{\sum_{i=1}^{T_n} \sum_{t=0}^{TST} q_{i,t}}\}$$
(3)

127 where WAi,t = water age at ith node at time step t; Tn = number of demand nodes of network; 128 Tstep = number of time steps into which total simulation time (TST) is divided; and qi,t = demand 129 requested at ith node at time step t. The three proposed formulations of Eqs. (1)–(3) represent 130 different ways to approach water quality evaluation. For example, with reference to DBP 131 formation, the use of Eq. (1) implies that more attention is given to the maximum concentration at 132 those nodes far from the disinfection points. The minimization of the mean water age [Eq. (2)] 133 considers the behaviour of the network in average, without controlling the extreme values. Finally, 134 Eq. (3) is based not only on the DBP concentrations but also takes into account the quantity of 135 users exposed to higher values. To provide recommendations on the selection of the most suitable 136 formulation, a comparison of performances of the three ObF1 formulations is presented.

137 The second objective function, ObF2, minimizes the number of valve closures (NoC):

$$138 \qquad ObF2 = \min\{NoC\} \tag{4}$$

*NoC* is defined as the number of valves to be closed to reroute the flow in the network. The aim of *ObF2* is to contain interventions in the network to reduce investment costs for placing new valves and to limit their movement. In fact, if only *ObF1* objective is considered, solutions with a huge number of valve operations may be generated, implying an unacceptable effort by the water utility. Moreover, the valves could be successively re-opened if required for a change of the system functioning. However, this may produce the releasing of accumulated material behind the closed section, aspect that is addressed by minimising the number of closures.

## 146 Decision variables and constraints

It is assumed that every pipe in the network has a potential shut-off valve. The decision variables in the optimization problem are the valves' status, represented at that stage by binary values (open or close) (Alfonso et al. 2010). Further investigations will consider the effects of percentages/degrees of valve closures or openings (Kang and Lansey 2009; Ostfeld and Salomons 2006).

The constraints are fixed considering that the operational status of the valves needs to guarantee the required service also in terms of pressure. Hence, the considered constraints are as follows: (1) any valve configuration status must guarantee the supply of water to all nodes, i.e., nodes cannot be disconnected; (2) the pressure Pi,t at each ith node at each time t should be within a fixed range:

 $156 \quad P_{min} < P_{i,t} < P_{max} \tag{5}$ 

# 157 **METHODOLOGY**

#### 158 *The procedures*

Twelve different procedures combining different optimization algorithms and formulations are compared (Table 1). The four algorithms used, described in detail in the following sections, are RS, LOC, and a combination of each of these two with AMGA2, a multiobjective evolutionary algorithm based on genetic algorithms. The first objective function is MaWA [Eq. (1)], MeWA [Eq. (2)], or DeMeWA [Eq. (3)], while the second objective function is always NoC [Eq. (4)]. The results are provided as Pareto fronts and maps to compare the different procedures. 165 **Table 1** Optimization procedures combining the *ObF1* formulations and the optimization algorithms.

166	PROCEDURE	ObF1	<b>OPTIMIZER</b>
167	P1	MaWA	RS
168	P2	MaWA	LOC
169	P3	MaWA	RS-AMGA2
170	P4	MaWA	LOC-AMGA2
170	P5	MeWA	RS
1/1	P6	MeWA	LOC
172	P7	MeWA	RS-AMGA2
173	P8	MeWA	LOC-AMGA2
174	P9	DeMeWA	RS
175	P10	DeMeWA	LOC
176	P11	DeMeWA	RS-AMGA2
177	P12	DeMeWA	LOC-AMGA2

## 178 Simulation setup

EPANET (Rossman 2000) is used as a WDN model for hydraulic and quality simulation (water
age evaluation). Since the aim of this paper is to present a new and general methodology to reduce
water age, at the present stage some simplifying hypotheses are considered:

- Even if in real WDN users are placed along pipes, demands are assumed to be concentrated in nodes. For the mean pipe length of the presented networks the corresponding approximation of water age is on the order of less than 1 s. Further investigations will consider demands distributed along pipes as in Farina et al. (2014) and Menapace et al. (2018).
- The pressure-driven approach is not used because the minimum pressure value in the constraint [Eq. (5)] is fixed in order to guarantee demand-driven functioning.

- Leakages are neglected even if they represent a component of demands. Their effect will
  be analyzed in future research.
- To verify the existence of disconnected nodes, a procedure implemented in EPANET is
  used. However, other methods could be adopted (e.g., Creaco et al. 2012).
- For water age evaluation complete mixing at nodes is assumed and dispersion is neglected.
   Although this assumption is questionable (Machell et al. 2009), its correction requires more
   complex computations, and for this reason they are still adopted in the majority of
   simulation tools and applications (Boccelli et al. 1998; Di Cristo and Leopardi 2008;
   Seyoum and Tanyimboh 2017).
- Input data uncertainty (Di Cristo et al. 2015) is not considered herein, but the same authors
   presented a robust optimization with respect to demand uncertainty in Marquez-Calvo et
   al. (2018).
- A standard model-based optimization framework, commonly used in the literature (e.g., Alfonso et al. 2010; Quintiliani et al. 2017), is adopted. An application compiled in C++ using the library of functions of the EPANET Programmer's Toolkit (Rossman 1999) was developed to set up the valve configurations in the input file and to run the hydraulic and water quality engines. The outputs of the application used by the optimization algorithm are ObF1 and ObF2 values.
- All objective functions are evaluated with respect to the original status of the network, i.e., with all valves open, corresponding to ObF2 = 0. This means that the "do-nothing" solution is always included in the Pareto front. In this way, a comparison is made on how much ObF1 improves for different configurations with respect to the original status.

# 210 **Optimization algorithms**

To describe the RS and LOC algorithms, the Class P network is defined as a network that has P pipes that can be closed through valve operation.

# 213 Random Search

214 Given a maximum number N of objective function evaluations and a maximum number P of valves

215 to close, M = N=P network con-figurations belonging to the same class are considered. The RS

216 algorithm generates M random network configurations for each class and selects the one with the

217 lowest ObF1. The procedure stops when all P classes have been analysed.

218

# 219

# 220 Loop for Optimal Valve Status Configuration

LOC is an algorithm specifically designed to solve the stated problem, which is based on procedures that find the best possible solution incrementally at each step, similarly to greedy algorithms (e.g., Alfonso et al. 2013; Banik et al. 2017a, b). As in the previous case, LOC is used to find P configurations of a network.

225 Starting from Class 0, corresponding to an initial condition where all valves of the network are 226 open, LOC investigates all possible configurations and selects the valve that produces the highest 227 ObF1 reduction in the entire network when it is closed. Then it is removed from the set of 228 "Remaining Valves" and added to the set of "Best Configurations." To set the second valve to 229 close, the algorithm considers the configurations with the valves previously closed, selecting 230 within the "Remaining Valves" set the valve that offers the ObF1 highest reduction. This valve is added to the "Best Configuration" set. The procedure stops when the P class has been reached. 231 232 LOC uses a predetermined, limited number of function evaluations to find a (sub optimal) Pareto 233 front. This number of evaluations is given by the expression:

$$Ne = \sum_{i=NP-P+1}^{NP} i \tag{6}$$

where *Ne* is the number of function evaluations, *NP* is the total number of pipes of the networkand *P* is the maximum number of valves to close.

#### 237 AMGA2

238 The AMGA2 by Tiwari et al. (2011) is a multiobjective evolutionary algorithm to find optimal 239 solutions. It is considered a steady-state genetic algorithm because its main Pareto front has a small 240 number of solutions, although other good solutions are stored in an archive. To produce the next 241 generation of populations, it uses all solutions in the main Pareto front mated with some of the 242 solutions in the archive. To decide which solutions to include in the new Pareto front, two criteria 243 are used: the degree of dominance of the solution and the diversity of the solution. In this way two 244 goals are reached, namely, a small number of function evaluations and the advantage of the 245 diversity of solutions in the archive. The good solutions that are not selected for the new Pareto

front are included in the archive. To maintain the archive, the solutions crowding a specific regionof the solution space are eliminated using the nearest-neighbour search strategy.

248 Some experiments, not reported in this paper, demonstrated that AMGA2 alone was not able to 249 find a satisfactory number of solutions because most of the generated networks were characterized 250 by disconnected nodes. To deal with this problem, Prasad and Walters (2006) modified their 251 algorithm to avoid the generation of networks with disconnections. In contrast, in this work the 252 search space is reduced to minimize the generation of networks with disconnected nodes by 253 combining AMGA2 with either RS or LOC (named RS-AMGA2 and LOC-AMGA2, 254 respectively). In this way, two objectives are met. First, some sets of candidate valves to be used 255 as decision variables by AMGA2 are generated, drastically reducing the search space. Second, a 256 reference initial population is given to AMGA2, improving its efficiency.

## 257 *Performance indicators*

In order to measure the improvement of RS and LOC algorithms by combining them with AMGA2, the following Index of Improvement (*IoI*) is used:

260 
$$Iol(F_k, F_j) = \frac{1}{|C(F_k, F_j)|} \sum_{c(F_k, F_j)} \frac{f_{j,m}^{(1)}}{f_{k,h}^{(1)}}$$
(7)

where  $F_k$  and  $F_j$  represent the solution of the Pareto fronts of AMGA2 (subscript k) and of each of its counterpart LOC or RS (subscript j), respectively, for a fixed value of *ObF2* (*NoC*). C is a set containing all the couples ( $F_k$ ,  $F_j$ ) and | $C(F_k$ ,  $F_j$ )| is its cardinality. Furthermore,  $f^{(1)}_{k,h}$  is the value of *ObF1* of the *h*-th tuple in the Pareto front k, and  $f^{(1)}_{j,m}$  is the value of *ObF1* of the *m*-th tuple in the Pareto front j.

In other words, considering a solution with the same number of operations NoC (ObF2), Eq. (7) estimates the ratio of the ObF1 value of the solution in the counterpart to the ObF1 value of the solution with AMGA2. The summation of all these ratios is divided by the number of solutions with the same ObF2 to consider a global value representing the efficiency of the procedures, regardless of the ObF1 formulation used. Then, the weighted average of the IoI (WAIoI) is evaluated:

272 
$$WAIol\left(\boldsymbol{F}_{k}, \boldsymbol{F}_{j}\right) = \frac{1}{\sum_{obF1} |\boldsymbol{c}(\boldsymbol{F}_{k(obF1)}, \boldsymbol{F}_{j(obF1)})|} \sum_{obF1} \left[ |\boldsymbol{c}(\boldsymbol{F}_{k(obF1)}, \boldsymbol{F}_{j(obF1)})| * Iol(\boldsymbol{F}_{k(obF1)}, \boldsymbol{F}_{j(obF1)}) \right]$$
273 (8)

where  $\sum_{ObF1}$  represents the summation of the sets *C* for all *ObF1* formulations.

To compare the performances of different ObF1 formulations, the differences between the initial condition values and the optimized ones of the following parameters are computed in each node:

277 
$$MaWA_i = max \{WA_t, \forall t = 0 \dots TST\}_i \quad \text{for } MaWA \text{ as } ObF1$$
(9)

278 
$$MeWA_{i} = \left(\frac{1}{T_{step}} \sum_{t=0}^{TST} WA_{t}\right)_{i} \qquad \text{for } MeWA \text{ as } ObF1 \qquad (10)$$

279 
$$DeMeWA_{i} = \left(\frac{\sum_{t=0}^{TST} WA_{t} \cdot q_{t}}{\sum_{t=0}^{TST} q_{t}}\right)_{i} \qquad \text{for } DeMeWA \text{ as } ObF1 \qquad (11)$$

In particular, MaWAi, MeWAi, and DeMeWAi = maximum, arithmetic mean, and demandweighted mean of ages observed at ith node during TST, respectively. A negative value of the differences between the initial condition values and the optimized ones, indicated as AMaWAi, AMeWai, and ADeMeWAi, means a re-duction of the age formulation value at the ith node.

To evaluate the quality of the solutions, the average (itt) and standard deviation (a) of the variations AMaWAi, AMeWAi, and ADeMeWAi observed in all nodes of the network are computed. Negative values of itt indicate an average reduction of the age in the network. A higher negative average indicates a better performance; a lower standard deviation indicates good homogeneity in the variation age in the network.

## 289 CASE STUDIES

290 Two distribution networks with different characteristics are selected to explore the performance of

the proposed procedures: Network PW06 by Prasad and Walters (2006) and Network J14 from the

292 database developed by the Kentucky Infrastructure Authority (Jolly et al. 2012).





Figure 1 Distribution networks schemes. (a) PW06 (Prasad and Walters 2006); (b) J14 (Jolly et al. 2012).

The PW06 network [Fig. 1(a)] has 47 pipes and 33 demand no-des, with elevations that vary between 10 and 30 m, and it is sup-plied from a single source (reservoir). The demands assigned in the nodes are the same as those in the original paper.

Network J14 [Fig. 1(b)] has the following characteristics: 377 demand nodes with elevations between 200 and 274 m, 3 tanks, 473 pipes with a total length of about 104 km, and 5 pump stations. The system is supplied from four sources, one at a head of 274 m and the others at around 200 m. In the schematization [Fig. 1(b)], while two sources are visible, the others are indicated as INLET 1 and INLET 2, located respectively at 12 and 62 km from the WDN. In all nodes, the same demand pattern is assigned, characterized by a 1-h time step multiplier with two picks of request around 10:00 a.m. and 9:00 p.m.

- 305 In both cases, the simulations were run long enough to guarantee stability of the hydraulic
- 306 conditions. The latter was achieved after 72 h of simulation for Network PW06 and 168 h of
- 307 simulation for Network J14.

# 308 ANALYSIS OF RESULTS AND DISCUSSION

309 The LOC algorithm requires a predefined number of evaluations, Ne [Eq. (6)]. In contrast, the

- 310 other algorithms do not use a predetermined Ne, which means that their performance depends
- 311 directly on the required function evaluations. The analysis of the performance is done considering
- 312 the fixed Ne of LOC as the baseline.



Figure 2 Results in terms of Pareto fronts for PW06 (Prasad and Walters 2006) (a, b, c) and J14 (Jolly et

al. 2012) (d, e, f). Procedures P1 to P4 (a, d); procedures P5 to P8 (b, e); procedures P9 to P12 (c, f).

As described in more detail in the following paragraphs, Fig. 2 shows the results of the procedures listed in Table 1 in terms of Pareto fronts for both case studies, while Table 2 reports the values of the indicator WAIoI [Eq. (8)] used to evaluate the performances of the optimization algorithms.

319	<b>Table 2</b> Values of WAIoI for both case studies.			
320	<b>Performance Indicator</b>	J14	<b>PW06</b>	
	WAIOI (FLOC-AMGA2, FLOC)	1.021	1.007	
321	WAIOI (FRS-AMGA2, FRS)	1.134	1.060	
322	WAIOI (FRS-AMGA2, FLOC)	1.010	1.022	
323				

## 324 **PW06** Network

- 325 In PW06 the required number of function evaluations is Ne = 425 [Eq. (6)] to obtain a 10-point
- 326 Pareto front. The values used as pressure thresholds in the constraint of Eq. (5), expressed in terms

327 of piezometric height, are Pmax = 100 m and Pmin = 10 m.

- 328 For PW06, the solutions reported in terms of Pareto fronts in Figs. 2(a-c) show that for all
- 329 considered ObF1 formulations, LOC generates a better front than that from RS. Moreover, RS and
- 330 RS-AMGA2 algorithms are able to find a limited number of solutions with respect to LOC and
- 331 LOC-AMGA2.
- 332 AMGA2 barely improves the Pareto front found by LOC. However, its improvement over RS is
- 333 significant. In fact, the use of AMGA2 in combination with RS makes it possible to reach the same
- 334 ObF1 values of RS by operating fewer valves. Moreover, this combination is also slightly better
- than LOC and LOC-AMGA2 solutions. This is confirmed by the WAIoI values reported in Table
- 336 2, which suggest that the addition of AMGA2 produces an improvement of 6.0% and 0.7% with
- 337 respect to the solutions of RS and LOC, respectively, while the Pareto front of RS-AMGA2 is
- about 2% better than the one from LOC.



#### 

Figure 3 Heat maps showing the frequency of valve closure from solutions of procedures P1 to P12 for network PW06 (Prasad and Walters 2006).

342	Fig. 3 represents for all procedures the heat maps showing the frequency of the valves included in
343	the solutions of the Pareto front; a darker dot indicates that the valve is more often considered. A
344	RS algorithm (P1-P5-P9) is characterized by the use of a large number of valves in the network,
345	which is not convenient in the operational context. The application of AMGA2 after RS (P3-P7-
346	P11) improves the solutions, focusing on only five or six valves to operate. LOC algorithm has

better behavior also without having to apply AMGA2 afterwards. Moreover, LOC and LOCAMGA2 consider almost the same valves, mainly placed on the largest diameters.

To compare the performances of different ObF1 formulations, the average ( $\mu$ ) and standard deviation ( $\sigma$ ) of the variation  $\Delta$ MaWai,  $\Delta$ MeWai, and  $\Delta$ DeMeWai for the optimized solutions obtained with LOC and LOC-AMGA2 for NoC = 5 are computed. This NoC number was selected considering that additional closures reduce ObF1 only marginally. For all cases, the obtained  $\mu$ values are negative, showing for all formulations a reduction in the average age with respect to the original condition. Insignificant differences have been observed among considered age formulations and between LOC and LOC-AMGA2 results.

The performance of each ObF1 is also estimated extracting the optimal network configurations and evaluating how well they performed for the remaining ObF1 formulations. It is observed that the use of each of the ObF1 formulations implies, on average, a reduction in the values of the other objective functions, when compared with the do-nothing option, almost reaching the values obtained when they are used as the optimization target.

## 361 J14 Network

For the J14 network, assuming that a maximum of 20 valves can be operated, the number of function evaluations, Ne, is 9270. The values used as pressure thresholds in the constraint of Eq. (5), expressed in terms of piezometric height, are Pmax = 100 m and Pmin = 10 m.

(5), expressed in terms of prezonietite height, are 1 max = 100 m and 1 mm = 10 m.

The Pareto fronts obtained for the J14 network are presented in Figs. 2(d–f), where the comparison among the different algorithms shows a similar tendency of what is obtained for the PW06 case.

367 In particular, LOC generates a better Pareto front than RS; AMGA2 improves slightly the solutions

of LOC, while those of RS are improved significantly. The WAIoI values (Table 2) indicate that

by adding AMGA2, LOC is improved by approximately 2% and RS by approximately 13%.

370 Finally, RS-AMGA2 produces an improvement of about 1% with respect to LOC.

371 In summary, the results suggest that the LOC algorithm produces a better Pareto front than RS.

Also, although the combination RS-AMGA2 works better than LOC, it requires more function

373 evaluations. The improvement that AMGA2 offers over LOC is negligible, whereas for RS it is

374 more significant.



375



P12 for network J14 (Jolly et al. 2012).

Fig. 4 shows the heat maps to provide a spatial indication of where and how frequently the pipes were selected by different procedures (Table 1). As expected, the solutions using the RS algorithm (P1, P5, and P9) do not focus on specific sectors of the network because the closures are randomly spread over the whole system. Independently of the selected ObF1, around 33% of the valves are included in at least one solution, meaning RS requires a large number of valves to be operated.

The solutions obtained with the RS-AMGA2, LOC, and LOC-AMGA2 algorithms are characterized by a reduced selection of valves to close, varying from 3% to 4.2% among all the possible decision variables. This confirms again that AMGA2 performs significantly better than 386 RS. A closer look at the valves selected in each experiment reveals that RS-AMGA2 individuates 387 different areas with respect to LOC and LOC-AMGA2. For the latter algorithms the considered 388 valves are concentrated in specific areas of the network involving mainly the larger diameters 389 located in the southern part of the system.

LOC

μ

12.96

-16.83

-25.97

**Formulation** 

*∆MaWA*<sub>i</sub>

*∆MeWA*<sub>i</sub>

 $\Delta DeMeWA_i$ 

391

392

393

394

395

**Table 3** Average ( $\mu$ ) and standard deviation ( $\sigma$ ) of the variations of *MaWA<sub>i</sub>*, *MeWA<sub>i</sub>* and *DeMeWA<sub>i</sub>*. in the J14 network (*NoC*=10).

σ

40.52

33.86

29.94

LOC-AMGA2

σ

38.14

33.86

29.94

μ

10.03

-16.83

-25.97

396 The average ( $\mu$ ) and standard deviation ( $\sigma$ ) of the variation  $\Delta$ MaWA ,  $\Delta$ MeWA , and  $\Delta$ DeMeWA 397 calculated between the in-itial values and those for the solutions of LOC and LOC-AMGA2 with 398 NoC = 10 are reported in Table 3. The NoC number has been again selected considering that 399 additional closures reduce ObF1 only marginally.  $\Delta$ MaWA has a positive  $\mu$ , indicating an average 400 increase of MaWA in the network, suggesting a bad performance of MaWA as ObF1. Both 401  $\Delta$ MeWA and  $\Delta$ DeMeWA have neg-ative  $\mu$  values and lower  $\sigma$  with respect to  $\Delta$ MaWA. 402  $\Delta DeMeWA$  is characterized by the highest negative average and the lowest stan-dard deviation, 403 which indicate its better performance as ObF1. No differences are observed between the LOC and 404 LOC-AMGA2 results.

405 Regarding the performance of the ObF1 formulations, extracting the optimal network 406 configurations and evaluating how well they performed for the remaining set of ObF1 not 407 se-lected, the results show mixed behaviors. Considering the configu-ration valve sets obtained 408 using MaWA as ObF1, this leads to almost no improvements for the other formulations with 409 respect to the case of NoC = 0. This has serious consequences for the maniporty of users, because 410 minimizing MaWA does not imply a dimin-ution of the residence time for a large part of the 411 WDN. The solutions obtained with MeWA do not modify the values of MaWA but improve those 412 of DeMeWA. This means that the majority of users would have a partial improvement, but not 413 those with high water residence time. Similarly, for the solution with DeMeWA, MaWA remains, 414 on average, near the zero-closure values regardless of the number of closures, while MeWA is 415 reduced to optimal levels. This means that most users would have access to water with a reduced416 age.

# 417 Performance of the LOC algorithm

To evaluate the performance of the LOC algorithm, its results are compared with the method proposed by Prasad and Walters (2006) and the brute-force search (BFS) procedure. Those tests were executed considering the PW06 network and fixing a constraint of 15 m as the minimum head in the network in accordance with the value used by Prasad and Walters (2006).



Figure 5 Comparison of LOC and Prasad and Walters (2006) solutions using MaWA (a), MeWA (b), and
DeMeWA (c).

425 A comparison of the results obtained by Prasad and Walters (2006) with those of LOC is shown 426 in Fig. 5. For the MaWA function, LOC finds several solutions that achieve a similar reduction in 427 water age with fewer pipe closures. Using the objective function MeWA [Fig. 5(b)], the LOC 428 solution with 9 closures is as good as the solution of Prasad and Walters (2006) with 11 closures. 429 For DeMeWA [Fig. 5(c)], LOC with 10 operations marginally dominates the solution by Prasad 430 and Walters (2006). Unfortunately, Prasad and Walters (2006) do not make any reference to the 431 number of evaluations required to obtain their results so the efficiency of the algorithms cannot be 432 compared.

A further experiment was designed to prove that the LOC method is suitable for finding a closeto-optimal solution. An exhaustive search of all solutions was carried out with a BFS in the smallest network, PW06, taking into account DeMeWA as ObF1. To reduce the execution time, an array of 28 CPU cores was used to perform the simulations in parallel. Both BFS and LOC were run for eight pipe closures to achieve the DeMeWA maximum reduction.

- 438 The solution found by BFS reduced the water age down to 2.8735 h, and it was available after 16.6
- 439 days of computational effort. Remarkably, the solution found by LOC reduced the water age down
- to 2.8736 h, requiring only 3 s. This demonstrates the efficiency of the proposed LOC algorithm.
- 441

 Table 4 Comparison between BFS and LOC solutions for the PW06 network.

	DeMeWA (hr) found		Number of simulations		Computational time (days)	
Number of closures ( <i>NoC</i> )					requ	ired
	BFS	LOC	BSF	LOC	BFS	LOC
1	4.1482	4.1482	4.70E+01	4.70E+01	4.73E-06	4.73E-06
2	3.8869	3.8869	1.07E+03	9.30E+01	1.07E-04	9.36E-06
3	3.6402	3.6402	1.55E+04	1.38E+02	1.56E-03	1.39E-05
4	3.3797	3.4119	1.61E+05	1.82E+02	1.62E-02	1.83E-05
5	3.1795	3.2528	1.28E+06	2.25E+02	1.29E-01	2.27E-05
6	3.0672	3.1072	8.02E+06	2.67E+02	8.07E-01	2.69E-05
7	2.9670	2.9670	4.03E+07	3.08E+02	4.06E+00	3.10E-05
8	2.8735	2.8736	1.65E+08	3.48E+02	1.66E+01	3.50E-05

442

443 To ensure the reliability of this comparison, the experiment was repeated considering different 444 pipe closures, from one to seven. The results are reported in Table 4. In all cases LOC performed 445 as well as BFS, with an advantage of several orders of magnitude in terms of computational time. 446 Unfortunately, it was not feasible to run BFS for NoC = 9, 10, and 11. Indeed, these would take 447 55, 145, and 299 days, respectively, because the required number of simulations are  $5.44 \times 108$ , 448  $1.44 \times 109$ , and  $2.97 \times 109$ , respectively. Moreover, when LOC runs for X closures, the solutions 449 for X - 1, X - 2.... 1 are immediately available, contrasting with BFS, which requires a separate 450 experiment for each number of closures.

## 451 **CONCLUSIONS**

The present paper compares the performances of 12 multiobjective optimization procedures to optimize valve management in WDNs for improving water quality, evaluated in terms of water age. The procedures derive from the combination of four different algo¬rithms (RS, LOC, RS-AMGA2, and LOC-AMGA2) and of three water quality objective function formulations (MaWA,

456 MeWA and DeMeWA). Two distribution networks of different complexity are considered.

457 The results show that the proposed LOC algorithm always pro-duces better solutions with respect

458 to RS, obtaining lower age val¬ues with the same number of closures. Moreover, heat maps show

459 that LOC considers candidate valves concentrated in specific areas of the network, which is an

advantage for operators. Its codification is very simple, and it produces a good compromisebetween the quality of the Pareto front and the required number of function evaluations.

The alternatives LOC-AMGA2 and RS-AMGA2 offer only a marginal improvement with respect to the solutions found by LOC, at the expense of having double function evaluations. This implies that, for this particular optimization problem, the LOC algorithm is the most convenient. The heat maps obtained with LOC show also that the operation on the larger pipes are more ef-ficient for the reduction of water age. The comparison of LOC with BFS demonstrates that, despite its simplicity, LOC achieves near-optimal results with very small computational effort, which justifies its use in large networks.

469 Regarding the comparison among the ObF1 formulations, the analysis of the average and standard 470 deviation of the variations  $\Delta$ MaWAi,  $\Delta$ MeWAi, and  $\Delta$ DeMeWAi observed in all nodes in-dicates 471 similar performances for the smaller Network PW06. For the more complex J14, the results 472 suggest better performances of MeWA and DeMeWA, indicating that the latter is the best one. 473 The evaluation of the different ObF1 shows that the miniminization of MaWA does not improve 474 MeWA and DeMeWA, meaning most water consumers would be affected at the expense of 475 improv-ing the water quality of a few. In conclusion, the use of averages, in particular the demand-476 weighted average, is recommended, because it would bring better water quality to most users.

477 DATA AVAILABILITY STATEMENT – The data of the models of WDNs analysed in the
478 paper, the complete results of the simulations and an executable file of the code generated to solve
479 the optimization problem are available from the corresponding author by request.

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#### 487 **REFERENCES**

- Alfonso, L., L. He, A. Lobbrecht, and R. Price. 2013. "Information theory applied to evaluate the
  discharge monitoring network of the Magdalena River." J. Hydroinf. 15 (1): 211–228.
  https://doi.org/10.2166/hydro .2012.066.
- Alfonso, L., A. Jonoski, and S. Dimitri. 2010. "Multiobjective optimization of operational
  responses for contaminant flushing in water distribution networks." J. Water Resour. Plann.
  Manage. 136 (1): 48–58. https://doi .org/10.1061/(ASCE)0733-9496(2010)136:1(48).
- Andrade, M. A., C. Y. Choi, K. Lansey, and D. Jung. 2016. "Enhanced artificial neural networks
  estimating water quality constraints for the optimal water distribution systems design." J. Water
  Resour. Plann. Manage. 142 (9): 04016024. https://doi.org/10.1061/(ASCE)WR .19435452.0000663.
- Banik, B. K., L. Alfonso, C. Di Cristo, and A. Leopardi. 2017a. "Greedy algorithms for sensor
  location in sewer systems." Water 9 (11): 856. https://doi.org/10.3390/w9110856.
- Banik, B. K., L. Alfonso, C. Di Cristo, A. Leopardi, and A. Mynett. 2017b. "Evaluation of different
  formulations to optimally locate sensors in sewer systems." J. Water Resour. Plann. Manage.
  143 (7): 04017026. https://doi.org/10.1061/(ASCE)WR.1943-5452.0000778.
- Bi, W., and G. Dandy. 2014. "Optimization of water distribution systems using online retrained
  metamodels." J. Water Resour. Plann. Manage. 140 (11): 04014032.
  https://doi.org/10.1061/(ASCE)WR.1943-5452 .0000419.
- Boccelli, D. L., M. E. Tryby, J. G. Uber, L. A. Rossman, M. L. Zierolf, and M. M. Polycarpou.
  1998. "Optimal scheduling of booster disinfection in water distribution systems." J. Water
  Resour. Plann. Manage. 124 (2): 99–111. https://doi.org/10.1061/(ASCE)0733-9496
  (1998)124:2(99).
- Carpentier, P., and G. Cohen. 1993. "Applied mathematics in water supply network management."
  Automatica 29 (5): 1215–1250. https://doi.org/10.1016/0005-1098(93)90048-X.
- 512 Castro Gama, M. E., Q. Pan, S. Salman, and A. Jonoski. 2015. "Multivariate optimization to
- 513 decrease total energy consumption in the water supply of abbiategrasso (Milan, Italy)." Environ.
- 514 Eng. Manage. J. 14 (9): 2019–2029.

- 515 Cembrano, G., G. Wells, J. Quevedo, R. P. Perez, and R. Argelaguet. 2000. "Optimal control of a
  516 water distribution network in a supervisory control system." Control Eng. Pract. 8 (10): 1177–
  517 1188. https://doi.org/10.1016/S0967-0661(00)00058-7.
- Cohen, D., U. Shamir, and G. Sinai. 2000a. "Optimal operation of multi-quality water supply
  systems. I: Introduction and the QC model." Eng. Optim. A35 32 (5): 549–584.
  https://doi.org/10.1080/03052150008 941313.
- 521 Cohen, D., U. Shamir, and G. Sinai. 2000b. "Optimal operation of multi-quality water supply 522 Eng. 687–719. systems. II: The QH model." Optim. A35 32 (6): 523 https://doi.org/10.1080/03052150008941318.
- 524 Creaco, E., S. Alvisi, and M. Franchini. 2015. "Multistep approach for optimizing design and
  525 operation of the C-town pipe network model." J. Water Resour. Plann. Manage. 142 (5):
  526 C4015005. https://doi.org/10.1061/(ASCE)WR.1943-5452.0000585.
- 527 Creaco, E., M. Franchini, and S. Alvisi. 2012. "Evaluating water demand shortfalls in segment
  528 analysis." Water Resour. Manage. 26 (8): 2301–2321. https://doi.org/10.1007/s11269-012529 0018-0.
- Di Cristo, C., and A. Leopardi. 2008. "Pollution source identification of accidental contamination
  in water distribution networks." J. Water Resour. Plann. Manage. 134 (2): 197–202.
  https://doi.org/10.1061 /(ASCE)0733-9496(2008)134:2(197).
- Di Cristo, C., A. Leopardi, and G. de Marinis. 2015. "Assessing measurements uncertainty on
  trihalomethanes prediction through kinetic models in water supply systems." J. Water Supply
  Res. Technol. AQUA 64 (5): 516–528. https://doi.org/10.2166/aqua.2014.036.
- Di Nardo, A., M. Di Natale, and G. F. Santonastaso. 2014. "A comparison between different
  techniques for water network sectorization." Water Sci. Technol. Water Supply 14 (6): 961–
  970. https://doi.org/10.2166/ws .2014.046.
- Farina, G., E. Creaco, and M. Franchini. 2014. "Using EPANET for modelling water distribution
  systems with users along the pipes." Civ. Eng. Environ. Syst. 31 (1): 36–50.
  https://doi.org/10.1080/10286608.2013.820279.

- Fu, G., Z. Kapelan, J. Kasprzyk, and P. Reed. 2013. "Optimal design of water distribution systems
  using many-objective visual analytics." J. Water Resour. Plann. Manage. 139 (6): 624–633.
  https://doi.org/10.1061/(ASCE)WR.1943-5452.0000311.
- Giustolisi, O., D. Laucelli, and L. Berardi. 2012. "Operational optimization: Water losses versus
  energy costs." J. Hydraul. Eng. 139 (4): 410–423. https://doi.org/10.1061/(ASCE)HY.19437900.0000681.
- Jamieson, D. G., U. Shamir, F. Martinez, and M. Franchini. 2007. "Conceptual design of a generic,
  real-time, near-optimal control system for water-distribution networks." J. Hydroinf. 9 (1): 3–
  14. https://doi.org/10.2166/hydro.2006.013.
- Jolly, M. D., A. D. Lothes, L. S. Bryson, and L. Ormsbee. 2012. "Research database of water
  distribution system models." J. Water Resour. Plann. Manage. 140 (4): 410–416.
  https://doi.org/10.1061/(ASCE)WR.1943 -5452.0000352.
- Jowitt, P. W., and G. Germanopoulos. 1992. "Optimal pump scheduling in water-supply
  networks." J. Water Resour. Plann. Manage. 118 (4): 406–422.
  https://doi.org/10.1061/(ASCE)0733-9496(1992)118:4(406).
- Kang, D., and K. Lansey. 2009. "Real-time optimal valve operation and booster disinfection for
  water quality in water distribution systems." J. Water Resour. Plann. Manage. 136 (4): 463–473.
  https://doi.org/10 .1061/(ASCE)WR.1943-5452.0000056.
- Kanta, L., E. Zechman, and K. Brumbelow. 2011. "Multiobjective evolutionary computation
  approach for redesigning water distribution systems to provide fire flows." J. Water Resour.
  Plann. Manage. 138 (2): 144–152. https://doi.org/10.1061/(ASCE)WR.1943-5452.0000156.
- Li, C., J. Z. Yu, T. Q. Zhang, X. W. Mao, and Y. J. Hu. 2015. "Multiob-jective optimization of
  water quality and rechlorination cost in water distribution systems." Urban Water J. 12 (8): 646–
  652. https://doi.org/10.1080/1573062X.2014.939093.
- Machell, J., and J. Boxall. 2014. "Modeling and field work to investigate the relationship between
  age and quality of tap water." J. Water Resour. Plann. Manage. 140 (9): 04014020.
  https://doi.org/10.1061/(ASCE) WR.1943-5452.0000383.

- Machell, J., J. Boxall, A. Saul, and D. Bramley. 2009. "Improved representation of water age in
  distribution networks to inform water quality." J. Water Resour. Plann. Manage. 135 (5): 382–
  391. https://doi.org/10.1061/(ASCE)0733-9496(2009)135:5(382).
- Mala-Jetmarova, H., N. Sultanova, and D. Savic. 2018. "Lost in optimisation of water distribution
  systems? A literature review of system design." Water (Switzerland) 10 (3): 307.
  https://doi.org/10.3390/w10030307.
- Marquez-Calvo, O., C. Quintiliani, L. Alfonso, C. Di Cristo, A. Leopardi, D. Solomatine, and G.
  de Marinis. 2018. "Robust optimization of valve management to improve water quality in WDNs
  under demand uncertainty." Urban Water J. 15 (10): 943–952. https://doi.org/10.1080
  /1573062X.2019.1595673.
- Menapace, A., D. Avesani, M. Righetti, A. Bellin, and G. Pisaturo. 2018. "Uniformly distributed
  demand EPANET extension." Water Resour. Manage. 32 (6): 2165–2180.
  https://doi.org/10.1007/s11269-018 -1924-6.
- Ostfeld, A., and E. Salomons. 2006. "Conjunctive optimal scheduling of pumping and booster
  chlorine injections in water distribution systems." Eng. Optim. 38 (03): 337–352.
  https://doi.org/10.1080 /03052150500478007.
- Prasad, T. D., and G. A. Walters. 2006. "Minimizing residence times by rerouting flows to improve
  water quality in distribution networks." Eng. Optim. 38 (8): 923–939.
  https://doi.org/10.1080/0305215060083 3036.
- Quintiliani, C., L. Alfonso, C. Di Cristo, A. Leopardi, and G. de Marinis. 2017. "Exploring the use
  of operational interventions in water distribution systems to reduce the formation of TTHMs."
  Procedia Eng. 186: 475–482. https://doi.org/10.1016/j.proeng.2017.03.258.
- Quintiliani, C., C. Di Cristo, and A. Leopardi. 2018. "Vulnerability assessment to trihalomethane
  exposure in water distribution systems." Water 10 (7): 912. https://doi.org/10.3390/w10070912.
- Rossman, L. A. 1999. "The EPANET programmer's toolkit for analysis of water distribution
  systems." In Proc., Annual Water Resources Planning and Management Conf., 1–10. Reston,
- 595 VA: ASCE.

- Rossman, L. A. 2000. EPANET 2: User's manual. EPA/600/R-00/057. Cincinnati: National Risk
  Management Research Laboratory, USEPA.
- Seyoum, A. G., and T. T. Tanyimboh. 2017. "Integration of hydraulic and water quality modelling
  in distribution networks: EPANET-PMX." Water Resour. Manage. 31 (14): 4485–4503.
  https://doi.org/10.1007 /s11269-017-1760-0.
- Shokoohi, M., M. Tabesh, S. Nazif, and M. Dini. 2017. "Water quality based multi-objective
  optimal design of water distribution systems." Water Resour. Manage. 31 (1): 93–108.
  https://doi.org/10.1007/s11269 -016-1512-6.
- Tiwari, S., G. Fadel, and K. Deb. 2011. "AMGA2: Improving the performance of the archivebased micro-genetic algorithm for multi-objective optimization." Eng. Optim. 43 (4): 377–401.
  https://doi.org/10.1080 /0305215X.2010.491549.
- Ulanicki, B., and P. R. Kennedy. 1994. "An optimization technique for water network operations
  and design." In Proc., 33rd Conf. on Decision and Control, 4114–4115. Piscataway, NJ: IEEE.
- 609 USEPA. 2002. Effects of water age on distribution system water quality. Washington, DC:610 USEPA.
- Zhao, Y., Y. J. Yang, Y. Shao, Y. Lee, and T. Zhang. 2018. "Demand-driven spatiotemporal
  variations of flow hydraulics and water age by comparative modeling analysis of distribution
  network." J. Water Resour. Plann. Manage. 144 (12): 04018074. https://doi.org/10.1061/(ASCE)
  WR.1943-5452.0000995.
- 615